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LETTER TO THE EDITOR

Lattice supersymmetry for $N=4$ Yang–Mills model

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Abstract. We introduce a supersymmetric four-dimensional $N=4$ Yang–Mills model on the space lattice. Our construction involves Dirac–Kähler formalism. We obtain the Hamiltonian on the space lattice with the correct classical continuum limit.

Recently the $N=4$ supersymmetric Yang–Mills (YM) theory has attracted a lot of attention. This is mainly motivated by the fact that it has been proved to be finite to all orders of perturbation theory (Mandelstam 1983, Brink *et al* 1983, Nomazie *et al* 1983, Howe *et al* 1983).

In the light of this development the lattice formulation of $N=4$ supersymmetric YM theory would be of interest, enabling, for instance, the non-perturbative study of the problem of finiteness of this model.

We choose the approach of Elitzur *et al* (1982) and work with the part of the supersymmetric algebra which involves only the Hamiltonian, that is

$$Q^2 = H. \tag{1}$$

This means automatically that we are working only on the space lattice (time continuous). We follow the Dirac–Kähler (DK) approach (Becher and Joos 1982, Joos 1984) modified to account for the fields in the adjoint representation (Aratyn *et al* 1984a). This note is a continuation of our work with the $N=2$ supersymmetric YM model on the lattice (Aratyn *et al* 1984b).

The general ansatz for describing fermionic degrees of freedom is:

$$\Psi = \sum_H \varphi(x, H)\, \mathrm{d}x^H \tag{2}$$

where H is an ordered set of up to four indices (Becher and Joos 1982).

More explicitly we work with the following differential form which describes our fermionic fields

$$\Psi = C_0 + \mathrm{i}A_\mu\, \mathrm{d}x^\mu - C_i\, \mathrm{d}x^0 \wedge \mathrm{d}x^i - \tfrac{1}{2}\varepsilon_{ijk}D_k\, \mathrm{d}x^i \wedge \mathrm{d}x^j - B_\mu\, \mathrm{d}x^\mu \vee \varepsilon - D_0\varepsilon \tag{3}$$

where ε is the volume element: $\varepsilon = \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3$, and $i, j, k = 1, 2, 3$. The $C_0(x), A_\mu(x) \ldots$ are real fields.

The 'spinor' transformation for the fermionic fields described by expression (3) in the DK formulation is discussed in detail by Becher and Joos (1982).

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The DK formalism establishes naturally the following correspondence between the four left-handed Weyl spinors and the components given in expression (3)†

$$\chi^{(i)}=\begin{pmatrix}\chi_1^{(i)}\\ \chi_2^{(i)}\end{pmatrix} \tag{4}$$

with

$$\begin{aligned}&\chi_1^{(1)}=C_0-\mathrm{i}D_0-C_3+\mathrm{i}D_3, &&\chi_1^{(2)}=-C_1+\mathrm{i}D_1+\mathrm{i}C_2+D_2,\\ &\chi_2^{(1)}=-C_1+\mathrm{i}D_1-\mathrm{i}C_2-D_2, &&\chi_2^{(2)}=C_0-\mathrm{i}D_0+C_3-\mathrm{i}D_3,\end{aligned} \tag{5}$$

$$\begin{aligned}&\chi_1^{(3)}=\mathrm{i}A_0+B_0-\mathrm{i}A_3-B_3, &&\chi_1^{(4)}=-\mathrm{i}A_1-B_1-A_2+\mathrm{i}B_2,\\ &\chi_2^{(3)}=-\mathrm{i}A_1-B_1+A_2-\mathrm{i}B_2, &&\chi_2^{(4)}=\mathrm{i}A_0+B_0+\mathrm{i}A_3+B_3.\end{aligned} \tag{5'}$$

This establishes a connection with the formulation of a $N=4$ SUSY YM model with a SU_4 structure (Brink *et al* 1977). In this formalism the 'scalar' fields are described by a sextuplet ϕ_{pr} of SU_4 with the condition:

$$\phi^{pra}=\tfrac{1}{2}\varepsilon^{prst}\phi^a_{st}=(\phi^a_{pr})^* \qquad prst=1,2,3,4 \tag{6}$$

'a' being an index of a non-Abelian gauge group SU_n.

Now let us introduce the following fields:

$$\begin{aligned}&P_1=\phi^{13}+\phi^{13*}, &&S_0=\mathrm{i}(\phi^{32}-\phi^{32*}),\\ &P_2=-\mathrm{i}(\phi^{13}-\phi^{13*}), &&S=\phi^{43}+\phi^{43*},\\ &P_3=\phi^{32}+\phi^{32*}, &&S_5=\mathrm{i}(\phi^{43}-\phi^{43*}).\end{aligned} \tag{7}$$

These scalar fields can conveniently be described by the following differential forms:

$$\begin{aligned}&W_1=S+\mathrm{i}S_0\,\mathrm{d}x^0+S_5\varepsilon\\ &W_2=\tfrac{1}{2}\varepsilon_{ijk}P_i\,\mathrm{d}x^0\wedge\mathrm{d}x^j\wedge\mathrm{d}x^k=P_i\,\mathrm{d}x^i\vee\varepsilon.\end{aligned} \tag{8}$$

Let us make the remark that the indices 0, 5 and i which appear in W_1 and W_2 are pure flavour indices and the corresponding matrices, obtained through the mapping $\mathrm{d}x^\mu\to\gamma^\mu$, act as flavour transformations on Ψ, when we construct the corresponding supersymmetric currents, without changing the scalar transformation properties of S, S_0 and P_i (for more detailed discussions see Joos 1984). This construction exhibits the geometrical nature of the scalar degrees of freedom on the space lattice: S and S_0 being site functions, S_5 a cube and P_i are connected with (j,k) plaquettes.

Therefore following the usual DK rules for substituting continuum derivatives by Δ_i^+, Δ_i^- derivatives

$$\begin{aligned}&\Delta_i^+f(x)=f(x+1)-f(x)\\ &\Delta_i^-f(x)=f(x)-f(x-1)\end{aligned}$$

we can propose the following supercharge for the free $N=4$ model (Abelian case):

$$Q=\sum_x J_0(x)$$

$$\begin{aligned}J_0=E^iA_i-H^iB_i-\varepsilon_{ijk}(\Delta_i^-P_j)C_k+(\Delta_k^+P_k)D_0-(\Delta_k^+S_0)C_k+(\Delta_k^+S)A_k\\ +(\Delta_k^-S_5)B_k+(\partial_0P_k)D_k-(\partial_0S_0)C_0+(\partial_0S)A_0+(\partial_0S_5)B_0\end{aligned} \tag{9}$$

† We use the γ matrices in the Weyl representation.

leading by (1) to the following Hamiltonian density (up to a normalisation factor):

$$\begin{aligned}\mathcal{H} = &\tfrac{1}{2}(E^{i2}+H^{i2})+\tfrac{1}{2}(\partial_0 P_k)(\partial_0 P_k)+\tfrac{1}{2}(\partial_0 S_0)(\partial_0 S_0)+\tfrac{1}{2}(\partial_0 S)(\partial_0 S)+\tfrac{1}{2}(\partial_0 S_5)(\partial_0 S_5)\\ &+\tfrac{1}{2}(\Delta_i^- P_j)(\Delta_i^- P_j)+\tfrac{1}{2}(\Delta_i^+ S_0)(\Delta_i^+ S_0)+\tfrac{1}{2}(\Delta_i^+ S)(\Delta_i^+ S)\\ &+\tfrac{1}{2}(\Delta_i^- S_5)(\Delta_i^- S_5)+\mathrm{i}A_0(\Delta_k^- A_k)+\mathrm{i}B_0(\Delta_k^+ B_k)-\mathrm{i}\varepsilon_{ijk}A_i(\Delta_j^- B_k)\\ &+\mathrm{i}C_0(\Delta_k^- C_k)+\mathrm{i}D_0(\Delta_k^+ D_k)-\mathrm{i}\varepsilon_{ijk}C_i(\Delta_j^- D_k)+E^i(\Delta_i^+ S)-H^i(\Delta_i^- S_5)\end{aligned} \tag{10}$$

where E^i, H^i are the usual electric and magnetic fields, ∂_0 being the time derivative. Because of ansatz (8) and the DK association of $\Delta^\pm$ derivatives to the different geometrical entities on the lattice, as expressed in (9), we obtain the right free Hamiltonian part for the fermions without fermion doubling problems.

We can add interaction to this model by substituting in (9) $\Delta^\pm$ by the corresponding 'covariant derivatives' on the lattice (4) (Aratyn *et al* 1984a), which are given by

$$\begin{aligned}\mathcal{D}_\mu^+\varphi(x,H) &= U(x,\mu)\varphi(x+e^\mu,H)-\varphi(x,H)U(x+e^H,\mu)\\ \mathcal{D}_\mu^-\varphi(x,H) &= \varphi(x,H)U^+(x+e^H-e^\mu,\mu)-U^+(x-e^\mu,\mu)\varphi(x-e^\mu,H)\end{aligned} \tag{11}$$

with $\varphi(x,H)$ given in (2), $U(x,\mu)$ being the link function which defines the non-Abelian gauge field SU_n; $e^H=e^{\mu_1}+e^{\mu_2}+\ldots+e^{\mu_A}$ for $H=\{\mu_1,\mu_2,\ldots,\mu_A\}$.

Therefore we replace (9) by the following gauge invariant expression†

$$\begin{aligned}J_0=\mathrm{Tr}\{&E^k(x)A_k^+(x)-H^k(x)B_k^+(x)-\varepsilon_{ijk}(\mathcal{D}_i^-P_j(x))C_k^+(x)\\ &+(\mathcal{D}_k^+P_k(x))D_0^+(x)-(\mathcal{D}_k^+S_0(x))C_k^+(x)+(\mathcal{D}_k^+S(x))A_k^+(x)\\ &+(\mathcal{D}_k^-S_5(x))B_k^+(x)+(\partial_0P_k(x))D_k^+(x)-(\partial_0S_0(x))C_0^+(x)\\ &+(\partial_0S(x))A_0^+(x)+(\partial_0S_5(x))B_0^+(x)\\ &+\mathrm{i}g[-C_0^+(x)(S_0(x)S(x)-S(x)S_0(x))\\ &+D_0^+(x)(S_0(x)S_5(x)-S_5(x)S_0(x+e_5))\\ &-B_0^+(x)(S(x)S_5(x)-S_5(x)S(x+e_5))\\ &+C_i^+(x+e_5-e^i)(P_i^+(x)S_5(x)-S_5(x+e_5-e^i)P^+(x+e_5))\\ &-D_i^+(x)(S(x)P_i(x)-P_i(x)S(x+e_5-e^i))\\ &-B_i^+(x)(S_0(x)P_i(x)-P_i(x)S_0(x+e_5-e^i))\\ &-\varepsilon_{ijk}B_i^+(x)P_j(x)U^+(x+e^k,i)P_k(x+e^k)U^+(x+e^j+e^k,i)]\}\end{aligned} \tag{12}$$

where $e_5=e^1+e^2+e^3$.

This zero component of the current is invariant under the gauge transformations (Aratyn *et al* 1984a)

$$\begin{aligned}\varphi(x,H)&\to\sigma(x)\varphi(x,H)\sigma^{-1}(x+e^H)\\ \varphi^+(x,H)&\to\sigma(x+e^H)\varphi^+(x,H)\sigma^{-1}(x).\end{aligned} \tag{13}$$

† Let us make the remark that the conjugation operation '+' introduced on the lattice appears as a consequence of the two possible different orientations on the lattice. This of course does not introduce any new degrees of freedom in the continuum limit.

Our supercharge is now

$$Q=\sum_x \tfrac{1}{2}(J_0(x)+J_0^+(x)). \tag{14}$$

In order to obtain the 'Hamiltonian' on the lattice we impose the following equal-time anticommutation relations for the fermion fields

$$\begin{aligned}
&\{A_\mu^{+ab}(x),\, A_\nu^{b'a'}(x')\}=\{B_\mu^{+ab}(x),\, B_\nu^{b'a'}(x')\}=2\delta_{aa'}\delta_{bb'}\delta_{\mu,\nu}\delta_{x,x'}\\
&\{D_i^{+ab}(x),\, D_j^{b'a'}(x')\}=\{C_i^{+ab}(x),\, C_j^{b'a'}(x')\}=2\delta_{aa'}\delta_{bb'}\delta_{ij}\delta_{x,x'}\\
&\{C_0^{+ab}(x),\, C_0^{b'a'}(x')\}=\{D_0^{+ab}(x),\, D_0^{b'a'}(x')\}=2\delta_{aa'}\delta_{bb'}\delta_{x,x'}
\end{aligned} \tag{15}$$

the other anticommutation relations being zero.

For the 'scalar' fields we impose the equal time commutation relations

$$\begin{aligned}
&[S_0^{ab}(x),\, \dot S_0^{b'a'+}(x')]=[S^{ab}(x),\, \dot S^{b'a'+}(x')]=[S_5^{ab}(x),\, \dot S_5^{b'a'+}(x')]=2\mathrm{i}\delta_{aa'}\delta_{bb'}\delta_{x,x'}\\
&[P_i^{ab}(x),\, \dot P_j^{b'a'+}(x')]=2\mathrm{i}\delta_{aa'}\delta_{bb'}\delta_{x,x'}\delta_{ij}
\end{aligned} \tag{15'}$$

the other commutation relations being zero.

The 'electric' and 'magnetic' fields on the lattice are defined by (Aratyn *et al* 1984a)

$$\begin{aligned}
&E_i(x)=\frac{\mathrm{i}}{ag}\frac{\partial U(x,i)}{\partial t}, \qquad E_i^+(x)=\frac{-\mathrm{i}}{ag}\frac{\partial U^+(x,i)}{\partial t}\\
&\mathscr{F}_{ij}=-\mathrm{i}a^2g\varepsilon_{ijk}H^k(x), \qquad \mathscr{F}_{ij}^+=\mathrm{i}a^2g\varepsilon_{ijk}H^{k+}(x)
\end{aligned} \tag{16}$$

with 'a' being the lattice spacing, and

$$\mathscr{F}_{\mu\nu}(x)=U(x,\mu)U(x+e^\mu,\nu)-U(x,\nu)U(x+e^\nu,\mu) \tag{17}$$

and the equal time commutation relations read as (Aratyn *et al* 1984a):

$$[\partial U_{aa'}^+(x,i)/\partial t,\, U_{bb'}(x',j)]=-2\mathrm{i}a^2g^2\delta_{ab'}\delta_{a'b}\delta_{ij}\delta_{xx'}. \tag{18}$$

We obtain the following Hamiltonian density

$$\mathscr{H}=\mathscr{H}_0+T_1+T_2+T_3+T_4+T_5 \tag{19}$$

where

$$\begin{aligned}
\mathscr{H}_0=\mathrm{Tr}\{&\tfrac14 E^i(x)E^{i+}(x)+\tfrac14 H^i(x)H^{i+}(x)+\tfrac14(\partial_0P_k(x))(\partial_0P_k^+(x))\\
&+\tfrac14(\partial_0S_0(x))(\partial_0S_0^+(x))+\tfrac14(\partial_0S(x))(\partial_0S^+(x))\\
&+\tfrac14(\partial_0S_5(x))(\partial_0S_5^+(x))+\tfrac14(\mathscr{D}_i^-P_j(x))(\mathscr{D}_i^-P_j(x))^+\\
&+\tfrac14(\mathscr{D}_i^+S_0(x))(\mathscr{D}_i^+S_0(x))^++\tfrac14(\mathscr{D}_i^+S(x))(\mathscr{D}_i^+S(x))^+\\
&+\tfrac14(\mathscr{D}_i^-S_5(x))(\mathscr{D}_i^-S_5(x))^++\tfrac12\mathrm{i}A_0(x)(\mathscr{D}_i^-A_i(x))^+\\
&+\tfrac12\mathrm{i}B_k^+(x)(\mathscr{D}_k^-B_0(x))-\tfrac12\mathrm{i}\varepsilon_{ijk}A_i^+(x)(\mathscr{D}_j^-B_k(x))\\
&+\tfrac12\mathrm{i}C_0(x)(\mathscr{D}_k^-C_k(x))^++\tfrac12\mathrm{i}D_0^+(x)(\mathscr{D}_j^+D_j(x))\\
&-\tfrac12\mathrm{i}\varepsilon_{ijk}D_j^+(x)(\mathscr{D}_i^+C_k(x))-\tfrac12gA_i^+(x)(A_i(x)S^+(x+e^i)-S^+(x)A_i(x))\\
&+\tfrac12g(B_0^+(x)S^+(x)-S^+(x+e_5)B_0^+(x))B_0(x)\\
&-\tfrac12g(S^+(x)C_0^+(x)-C_0^+(x)S^+(x))C_0(x)
\end{aligned}$$

$$
\begin{aligned}
&+\tfrac{1}{2}gD_i^+(x)(S(x)D_i(x)-D_i(x)S(x+e_5-e^i))\\
&+\tfrac{1}{2}g(S_5^+(x)A_0^+(x)-A_0^+(x+e_5)S_5^+(x))B_0(x)\\
&-\tfrac{1}{2}gB_i(x)(A_i(x+e_5-e^i)S_5^+(x)-S_5^+(x-e^i)A_i(x-e^i))\\
&+\tfrac{1}{2}g(S_5^+(x)C_0^+(x)-C_0^+(x+e_5)S_5^+(x))D_0(x)\\
&-\tfrac{1}{2}gC_i^+(x+e_5-e^i)(D_i^+(x)S_5(x)-S_5(x+e_5-e^i)D_i^+(x+e_5))\\
&+\tfrac{1}{2}g(A_0^+(x)S_0^+(x)-S_0^+(x)A_0^+(x))C_0(x)\\
&+\tfrac{1}{2}gA_i^+(x)(C_i(x)S_0^+(x+e^i)-S_0^+(x)C_i(x))\\
&-\tfrac{1}{2}g(B_0^+(x)S_0^+(x)-S_0^+(x+e_5)B_0^+(x))D_0(x)\\
&+\tfrac{1}{2}g(D_i^+(x)S_0^+(x)-S_0^+(x+e_5-e^i)D_i^+(x))B_i(x)\\
&-\tfrac{1}{2}g\varepsilon_{ijk}A_i^+(x)(C_k^+(x-e^k)P_j(x-e^k)-P_j(x)C_k^+(x+e^i))\\
&-\tfrac{1}{2}gA_i^+(x)(D_0(x)P_i^+(x+e^i)-P_i^+(x-e_5+e^i)D_0(x-e_5+e^i))\\
&+\tfrac{1}{2}g(P_i^+(x)A_0^+(x)-A_0^+(x+e_5-e^i)P_i^+(x))D_i(x)\\
&-\tfrac{1}{2}g(B_0^+(x)P_i(x)-P_i(x+e_5)B_0^+(x+e_5-e^i))C_i(x+e_5-e^i)\\
&-\tfrac{1}{2}g(P_i^+(x)C_0^+(x)-C_0^+(x+e_5-e^i)P_i^+(x))B_i(x)\\
&+\tfrac{1}{2}g\varepsilon_{ijk}U(x+e^j+e^k,i)D_k^+(x+e^k)U(x+e^k,i)P_j^+(x)B_i(x)\\
&+\tfrac{1}{2}g\varepsilon_{ijk}U(x+e^j+e^k,i)P_k^+(x+e^k)U(x+e^k,i)D_j^+(x)B_i(x)\\
&+\tfrac{1}{4}g^2[(S^+(x)S_0^+(x)-S_0^+(x)S^+(x))(S_0(x)S(x)-S(x)S_0(x))\\
&+(S_5^+(x)S_0^+(x)-S_0^+(x+e_5)S_5^+(x))(S_0(x)S_5(x)-S_5(x)S_0(x+e_5))\\
&+(S_5^+(x)S^+(x)-S^+(x+e_5)S_5^+(x))(S(x)S_5(x)-S_5(x)S(x+e_5))\\
&+(S_5^+(x)P_i(x)-P_i(x+e_5)S_5^+(x+e_5-e^i))\\
&\times(P_i^+(x)S_5(x)-S_5(x+e_5-e^i)P_i^+(x+e_5))\\
&+(P_i^+(x)S^+(x)-S^+(x+e_5-e^i)P_i^+(x))\\
&\times(S(x)P_i(x)-P_i(x)S(x+e_5-e^i))\\
&+(P_i^+S_0^+(x)-S_0^+(x+e_5-e^i)P_i^+(x))\\
&\times(S_0(x)P_i(x)-P_i(x)S_0(x+e_5-e^i))\\
&+\varepsilon_{ijk}\varepsilon_{lmn}U(x+e^j+e^k,i)P_k^+(x+e^k)U(x+e^k,i)P_j^+(x)P_m(x)\\
&\times U^+(x+e^n,l)P_n(x+e^n)U^+(x+e^m+e^n,l)]\\
&+\tfrac{1}{2}g^2\varepsilon_{ijk}U(x+e^j+e^k,i)P_k^+(x+e^k)U(x+e^k,i)P_j^+(x)\\
&\times(S_0(x)P_i(x)-P_i(x)S_0(x+e_5-e^i))+\mathrm{HC}\},
\end{aligned}\tag{20}
$$

$$
\begin{aligned}
T_1=\mathrm{Tr}[&\tfrac{1}{2}\mathrm{i}g(P_i^+(x)S^+(x)-S^+(x+e_5-e^i)P_i^+(x))\partial_0P_i(x)\\
&-\tfrac{1}{2}\mathrm{i}g(S^+(x)S_0^+(x)-S_0^+(x)S^+(x))\partial_0S_0(x)\\
&+\tfrac{1}{2}\mathrm{i}g(S_5^+(x)S^+(x)-S^+(x+e_5)S_5^+(x))\partial_0S_5(x)\\
&+\tfrac{1}{2}E^i(x)(\mathscr{D}_i^+S(x))^+-\tfrac{1}{2}H^i(x)(\mathscr{D}_i^-S_5(x))^++\mathrm{HC}],
\end{aligned}\tag{21}
$$

$$T_2 = -\tfrac{1}{4}\mathrm{i}ag^2\varepsilon_{ijk}\,\mathrm{Tr}[B_i^+(x)P_j(x)U^+(x+e^k, i)P_k(x+e^k)A_i^+(x+e^j+e^k)$$
$$+B_i^+(x)P_j(x)A_i^+(x+e^k)P_k(x+e^k)U^+(x+e^j+e^k, i]+\mathrm{HC}, \qquad (22)$$
$$T_3 = \mathrm{Tr}[\tfrac{1}{2}\mathrm{i}g(P_i(x)S_0^+(x) - S_0^+(x+e_5-e^i)P_i^+(x))H^i(x)$$
$$+\tfrac{1}{2}\varepsilon_{ijk}(\mathscr{D}_i^- P_j(x))(\mathscr{D}_k^+ S_0(x))^+ + \mathrm{HC}], \qquad (23)$$
$$T_4 = \mathrm{Tr}[-\tfrac{1}{4}(\mathscr{D}_i^- P_j(x))(\mathscr{D}_j^- P_i(x))^+ + \tfrac{1}{4}(\mathscr{D}_i^+ P_i(x))(\mathscr{D}_j^+ P_j(x))^+$$
$$-\tfrac{1}{2}\mathrm{i}g\varepsilon_{ijk}U(x+e^j+e^k, i)P_k^+(x+e^k)U(x+e^k, i)P_j^+(x)H^i(x) + \mathrm{HC}], \qquad (24)$$
$$T_5 = \mathrm{Tr}[\tfrac{1}{2}\mathrm{i}g\varepsilon_{ijk}(S_5^+(x)P_k(x) - P_k(x+e_5)S_5^+(x+e_5-e^k))(\mathscr{D}_i^- P_j(x+e_5-e^k))$$
$$-\tfrac{1}{2}\mathrm{i}g(S_5^+(x)S_0^+(x) - S_0^+(x+e_5)S_5^+(x))(\mathscr{D}_k^+ P_k(x))$$
$$+\tfrac{1}{2}\mathrm{i}g(S_5^+(x)P_i(x) - P_i(x+e_5)S_5^+(x+e_5-e^i))(\mathscr{D}_i^+ S_0(x+e_5-e^i))$$
$$+\tfrac{1}{2}\mathrm{i}g(P_i^+(x)S_0^+(x) - S_0^+(x+e_5-e^i)P_i^+(x))(\mathscr{D}_i^- S_5(x))$$
$$+\tfrac{1}{2}\mathrm{i}g\varepsilon_{ijk}U(x+e^j+e^k, i)P_k^+(x+e^k)U(x+e^k, i)(\mathscr{D}_i^- S_5(x)) + \mathrm{HC}]. \qquad (25)$$

Now, $\mathscr{H}_0 + T_1$, after using integration by parts and applying the Gauss law and the Bianchi identities, gives the Hamiltonian which in the continuum limit tends to the correct one. The extra terms, T_2, T_3, T_4 and T_5 will disappear in the continuum limit but their presence is necessary in order to maintain the invariance under the supersymmetric transformation on the lattice generated by Q given in (14).

In conclusion we have succeeded in formulating the supersymmetric $N=4$ YM theory on the space lattice having the correct tree level continuum limit.

A complete discussion of the $N=4$ supersymmetry algebra within the DK framework is under preparation. We also intend to discuss the invariance of the action defined on the lattice and the corresponding 'conserved' currents.

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